UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/13 Paper 1, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Qu No	Commentary	Solution	Marks	Part Mark	Total
1	Finds partial fractions.	$\dfrac{1}{r(r+2)}=\dfrac{1}{2}\left\{\dfrac{1}{r}-\dfrac{1}{r+2}\right\}$	M1A1		
		$\displaystyle\sum_{r=1}^{n}\frac{1}{r(r+2)}=$			
	Use method of differences.	$\dfrac{1}{2}\left\{\left[\dfrac{1}{n}-\dfrac{1}{n+2}\right]+\left[\dfrac{1}{n-1}-\dfrac{1}{n+1}\right]+\ldots+\left[\dfrac{1}{2}-\dfrac{1}{4}\right]+\left[1-\dfrac{1}{3}\right]\right\}$	M1		
	Obtains results.	$=\dfrac{1}{2}\left\{\dfrac{3}{2}-\dfrac{1}{n+1}-\dfrac{1}{n+2}\right\}$ (acf) $\Rightarrow S_\infty=\dfrac{3}{4}$	A1A1√	5	[5]
2	(States proposition.)	$(\mathrm{P}_n:\ u_n=4\left(\dfrac{3}{4}\right)^n-2)$			
	Proves base case.	Let $n=1$ $\quad 4\times\dfrac{3}{4}-2=3-2=1\Rightarrow \mathrm{P}_1$ true.	B1		
	States Inductive hypothesis.	Assume P_k is true for some k.	B1		
	Proves inductive step.	$u_{k+1}=\dfrac{3\left\{4\left(\dfrac{3}{4}\right)^k-2\right\}-2}{4}=4.\dfrac{3}{4}.\left(\dfrac{3}{4}\right)^k-\dfrac{6+2}{4}$	M1		
		$=4.\left(\dfrac{3}{4}\right)^{k+1}-2\quad\therefore \mathrm{P}_k\Rightarrow \mathrm{P}_{k+1}$	A1		
	States conclusion.	$\therefore$ By PMI P_n is true $\forall$ positive integers.	A1	5	[5]
3		$y+x\dfrac{\mathrm{d}y}{\mathrm{d}x}+3(x+y)^2\left(1+\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=0$	B1B1		
		$0+y'+3+3y'=0$			
		$\Rightarrow y'=-\dfrac{3}{4}$ (AG)	B1	3	
		$\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{\mathrm{d}y}{\mathrm{d}x}+x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+6(x+y)\left(1+\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3(x+y)^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$	B1 B1B1		
		$-\dfrac{3}{4}-\dfrac{3}{4}+y''+6\times\dfrac{1}{16}+3y''=0$	M1		
		$\Rightarrow y''=\dfrac{9}{32}$	A1	5	
		N.B. Mark similarly if expression expanded before differentiating.			[8]

Qu No	Commentary	Solution	Marks	Part Mark	Total
4		$I_n = \int_1^e x^2(\ln x)^n \, dx$			
	Integrates by parts.	$= \left[(\ln x)^n \frac{x^3}{3}\right]_1^e - \int_1^e n(\ln x)^{n-1} \cdot \frac{1}{x} \cdot \frac{x^3}{3} dx$	M1A1 A1		
	Obtains reduction formula.	$= \frac{e^3}{3} - \frac{n}{3} I_{n-1}$ (AG)	A1	4	
	Finds I_0 (or I_1)	$I_0 = \int_1^e x^2 dx = \left[\frac{x^3}{3}\right]_1^e = \frac{e^3 - 1}{3}$	B1		
	and uses reduction formula. (M1A1 for I_1 if found immediately.)	$\Rightarrow I_1 = \frac{e^3}{3} - \frac{1}{3}\left(\frac{e^3 - 1}{3}\right) = \frac{2e^3 + 1}{9}$	M1		
	Obtains I_2 .	$\Rightarrow I_2 = \frac{e^3}{3} - \frac{2}{3}\left(\frac{2e^3 + 1}{9}\right) = \frac{5e^3 - 2}{27}$	A1		
	Obtains I_3 .	$\Rightarrow I_3 = \frac{e^3}{3} - \left(\frac{5e^3 - 2}{27}\right) = \frac{4e^3 + 2}{27}$	A1	4	[8]
5	Proves initial result.	$(\mathbf{A} + k\mathbf{I})\mathbf{e} = \mathbf{Ae} + k\mathbf{Ie} = \lambda\mathbf{e} + k\mathbf{e} = (\lambda + k)\mathbf{e}$ $\therefore$ $(\mathbf{A} + k\mathbf{I})$ has eigenvalue $(\lambda + k)$ with corresponding eigenvector $\mathbf{e}$.	M1A1	2	[2]
	Finds eigenvectors corresponding to given eigenvalues.	Eigenvalues are –3 and 4 (given). Corresponding eigenvectors are $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	M1A1 A1	3	[3]
	Finds corresponding eigenvalue.	Third eigenvalue is 6.	B1	1	[1]
	Recognises the result proved initially.	$\mathbf{C} = \mathbf{B} - 3\mathbf{I}$ (Stated or implied.)	M1		
	Gives eigenvalues	Eigenvalues: –6, 1, 3 .	A1		
	and matches eigenvectors.	Corresponding eigenvectors are: $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$. (OE) (For 'non hence' method, using characteristic equation, award B1 rather than M1A1 for eigenvalues, followed by B1 for eigenvectors.)	A1√	3	[3]

Qu No	Commentary	Solution	Marks	Part Marks	Total
6	States vertical asymptote.	Vertical asymptote is $x = 2$.	B1		
	Finds oblique asymptote.	$y = x + 2 + \dfrac{4}{x-2}$ Oblique asymptote is $y = x + 2$.	M1 A1	3	
	Differentiates and equates to zero.	$y' = 1 - \dfrac{4}{(x-2)^2} = 0 \Rightarrow (x-2)^2 = 4$	M1		
	Finds x coordinates.	$x = 0, 4$.	A1		
	States coordinates of turning points.	Turning points are (0,0) and (4,8)	A1	3	
	Deduct at most 1 mark for poor forms at infinity.	Axes and both asymptotes correct. Upper branch correct. Lower branch correct.	B1 B1 B1	3	[9]
7	Complete strategy and getting halfway.	$\left(z+\dfrac{1}{z}\right)^4\left(z-\dfrac{1}{z}\right)^2 = \left(z^2+2+\dfrac{1}{z^2}\right)\left(z^4-2+\dfrac{1}{z^4}\right)$	M1A1		
	Fully correct.	$= z^6 + 2z^4 - z^2 - 4 - \dfrac{1}{z^2} + \dfrac{2}{z^4} + \dfrac{1}{z^6}$	A1		
	Grouping.	$= \left(z^6 + \dfrac{1}{z^6}\right) + 2\left(z^4 + \dfrac{1}{z^4}\right) - \left(z^2 + \dfrac{1}{z^2}\right) - 4$	M1		
	Correct LHS and RHS.	$16\cos^4\theta \cdot -4\sin^2\theta = 2\cos 6\theta + 4\cos 4\theta - 2\cos 2\theta - 4$ $64\cos^4\theta\sin^2\theta = 4 + 2\cos 2\theta - 4\cos 4\theta - 2\cos 6\theta$	A1(L) A1(R)	6	
		$x = 2\cos\theta \quad \dfrac{dx}{d\theta} = -2\sin\theta$ $x = 1 \Rightarrow \theta = \dfrac{\pi}{3} \quad x = 2 \Rightarrow \theta = 0$			
	Sets up substitution.	$-\int_{\frac{\pi}{3}}^{0} 16\cos^4\theta.4\sin^2\theta d\theta = \int_0^{\frac{\pi}{3}} 64\cos^4\theta\sin^2\theta d\theta$ (LR – allow use of 2π, if seen.).	M1		
	Uses result obtained above.	$= \int_0^{\frac{\pi}{3}} (4 + 2\cos 2\theta - 4\cos 4\theta - 2\cos 6\theta) d\theta$	M1		
		$= \left[4\theta + \sin 2\theta - \sin 4\theta - \dfrac{1}{3}\sin 6\theta\right]_0^{\frac{\pi}{3}}$	A1		
	Obtains result.	$= \left[\dfrac{4\pi}{3} + \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2}\right] = \dfrac{4\pi}{3} + \sqrt{3}$ (AG)	A1	4	[10]

Qu No	Commentary	Solution	Marks	Part Mark	Total
8 (i)	Deduces initial result.	$u=-\alpha+\beta+\gamma \Rightarrow u+2\alpha=\alpha+\beta+\gamma=1$	M1A1		
	Substitutes into cubic equation.	$\Rightarrow \alpha=\left(\dfrac{1-u}{2}\right)$	M1		
	Deduces new cubic equation.	$\Rightarrow \left(\dfrac{1-u}{2}\right)^3-\left(\dfrac{1-u}{2}\right)^2-3\left(\dfrac{1-u}{2}\right)-10=0$	A1		
		$\Rightarrow \ldots \Rightarrow u^3-u^2-13u+93=0$	A1	5	
(ii)	Deduces initial result.	$\alpha\beta\gamma=10$	B1		
		$\Rightarrow v=\dfrac{1}{\beta\gamma} \Rightarrow \dfrac{v}{\alpha}=\dfrac{1}{\alpha\beta\gamma}=\dfrac{1}{10} \Rightarrow \alpha=10v$	M1A1		
	Substitutes into cubic equation.	$(10v)^3-(10v)^2-3(10v)-10=0$	M1		
	Deduces new cubic equation.	$\Rightarrow 100v^3-10v^2-3v-1=0$	A1	5	[10]
	Alternatively:				
8 (i)	**For final 3 marks in (i):**	Let equation be $u^3+bu^2+cu+d=0$.			
	Award M1 for an attempt at formulae for all three coefficients. A1 for any two correct. A1 for completion	$-b=\sum\alpha=1 \Rightarrow b=-1$ $c=4\sum\alpha\beta-(\sum\alpha)^2$ $=4\times(-3)-1^2=-13$ $-d=4\sum\alpha\sum\alpha\beta-\left(\sum\alpha\right)^3-8\alpha\beta\gamma$ $=4\times1\times(-3)-1^3-8\times10=-93$ So $u^3-u^2-13u+93=0$		(5)	
(ii)	**For final 4 marks in (ii):** Award M1 for an attempt at formulae for all three coefficients. A1 for any one correct. A1 for a second one correct. A1 for completion.	Let equation be $v^3+bv^2+cv+d=0$. $-b=\dfrac{\Sigma\alpha}{\alpha\beta\gamma}=\dfrac{1}{10} \Rightarrow b=-\dfrac{1}{10}$ $c=\dfrac{\Sigma\alpha\beta}{(\alpha\beta\gamma)^2}=\dfrac{-3}{10^2}=-\dfrac{3}{100}$ $-d=\dfrac{1}{(\alpha\beta\gamma)^2}=\dfrac{1}{10^2} \Rightarrow d=-\dfrac{1}{100}$ So $v^3-\dfrac{1}{10}v^2-\dfrac{3}{100}v-\dfrac{1}{100}=0$ or $100v^3-10v^2-3v-1=0$.		(5)	[10]

Qu No	Commentary	Solution	Marks	Part Mark	Total
9	Finds normal vector to plane.	$\begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ 1 & -2 & -1 \\ 1 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ 1 \\ 4 \end{pmatrix}$ or $\begin{matrix} x = 2+\lambda+\mu \\ y = -3-2\lambda+2\mu \\ z = 1-\lambda-2\mu \end{matrix}$	M1A1		
	Uses known point to find constant term.	Plane equation is $6x + y + 4z$ = constant. Substitute $(2, -3, 1) \Rightarrow 12 - 3 + 4 = 13$. Or eliminate λ and μ. $\Rightarrow \Pi_1 : 6x + y + 4z = 13$	M1 A1	4	
	Angle between normals is equal to angle between planes.	$\cos\theta = \dfrac{(6\mathrm{i}+\mathrm{j}+4\mathrm{k}).(3\mathrm{i}-2\mathrm{j}-3\mathrm{k})}{\sqrt{6^2+1^2+4^2}\sqrt{3^2+2^2+3^2}} = \dfrac{4}{\sqrt{53}\sqrt{22}}$ $\Rightarrow \theta = 83.3°$ or 1.45 rad.	M1A1 A1	3	
	Solve plane equation simultaneously.	$6x + y + 4z = 13$ and $3x - 2y - 3z = 4$ Obtains e.g. $y + 2z = 1$ and $3x + z = 6$ Or two of $(0,-11,6)$, $(11/6,0,1/2)$, $(2,1,0)$	M1 A1A1		
	(Note: may find direction from vector product and use with one point.)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ -6 \\ 3 \end{pmatrix}$ (OE)	A1	4	
		Alternatively: Direction of line from vector product. Finds a point on line. States equation of line.	 (M1A1) (A1) (A1)		[11]

Qu No	Commentary	Solution	Marks	Part Mark	Total
10	Reduces augmented matrix	$\begin{pmatrix} 1 & -2 & -2 & -7 \\ 2 & a-9 & -10 & -11 \\ 3 & -6 & 2a & -29 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & -2 & -7 \\ 0 & a-5 & -6 & 3 \\ 0 & 0 & 2a+6 & -8 \end{pmatrix}$	M1A1		
	Obtains set of values for a, giving unique solutions.	Unique solution for all real a except $a = -3$ or 5	A1A1	4	
		Alternatively for first two marks: $\begin{vmatrix} 1 & -2 & -2 \\ 2 & (a-9) & -10 \\ 3 & -6 & 2a \end{vmatrix} \neq 0 \Rightarrow (a-5)(a+3) \neq 0$			
(i)	Case of no solutions.	$a = -3 : 2a + 6 = 0$ $\Rightarrow 0z = -8 \Rightarrow$ no solutions (AG)	M1A1√	2	
(ii)	Case of infinite solutions.	$a = 5 : \Rightarrow z = -\frac{1}{2}$ and $x - 2y = -8$ (*) $\therefore$ infinite number of solutions (AG)	M1A1√	2	
	Obtains particular solution.	$z = -\frac{1}{2}$ and $x + y + z = 2 \Rightarrow x + y = \frac{5}{2}$	B1		
		Solving simultaneously with (*) gives			
		$x = -1 \quad y = \frac{7}{2} \quad z = -\frac{1}{2}$	M1A1	3	[11]

Qu No	Commentary	Solution	Marks	Part Mark	Total
11	**EITHER**				
	Uses $x^2 + y^2 = r^2$, $x = r\cos\theta$ and $y = r\sin\theta$.	$(x^2+y^2)^2 = a^2(x^2-y^2)$ $\Rightarrow r^2 = a^2\left(\frac{x^2}{r^2} - \frac{y^2}{r^2}\right)$	M1		
		$= a^2(\cos^2\theta - \sin^2\theta) = a^2\cos 2\theta$ (AG)	A1	2	
	One mark for each loop, or half of whole curve.	Sketches C.	B2,1,0	2	
	Uses sector area formula.	Area $= \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} a^2\cos 2\theta \, d\theta \left(= \int_0^{\frac{\pi}{4}} a^2 \cos 2\theta \, d\theta\right)$	M1		
	S.C. Omission of ½ factor, but correct integration gets B1.	$= a^2\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}} = \frac{a^2}{2}$	A1A1	3	
	Differentiates.	$2(x^2+y^2)(2x+2yy') = a^2(2x-2yy')$	B1B1		
	Puts $y' = 0$.	$y' = 0 \Rightarrow 2x(x^2+y^2) = a^2x$	M1		
	Obtains coordinates.	$\Rightarrow 2r^2 = a^2 \Rightarrow r = \frac{a}{\sqrt{2}}$ $(r \geq 0)$	A1		
		$\Rightarrow \cos 2\theta = \frac{1}{2}$	M1		
		$\Rightarrow \theta = \pm\frac{\pi}{6}, \pm\frac{5\pi}{6}$	A1A1	7	
		i.e. $\left(\frac{a}{\sqrt{2}}, \pm\frac{\pi}{6}\right)$ and $\left(\frac{a}{\sqrt{2}}, \pm\frac{5\pi}{6}\right)$			[14]

Qu No	Commentary	Solution	Marks	Part Mark	Total
	Alternatively for last 7 marks:				
	Obtains condition for tangent parallel to initial line.	$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta}=0 \Rightarrow r\cos\theta+\frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta=0$	(M1A1)		
	Differentiates equation of *C*.	$2r\frac{\mathrm{d}r}{\mathrm{d}\theta}=-2a^2\sin 2\theta \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}\theta}=-\frac{a^2}{r}\sin 2\theta$	(A1)		
	Forms equation.	$\therefore r\cos\theta-\frac{a^2}{r}\sin 2\theta\sin\theta=0$	(M1)		
		$\therefore a^2\cos 2\theta\cos\theta=a^2\sin 2\theta\sin\theta$			
		$\therefore \frac{1}{\tan 2\theta}=\tan\theta$			
		$\therefore \frac{1-t^2}{2t}=t \Rightarrow 2t^2=1-t^2 \Rightarrow 3t^2=1$	(M1)		
	Solves for tan*θ*,	$\therefore t=\pm\frac{1}{\sqrt{3}}$			
	and *θ*.	$\therefore \theta=\pm\frac{\pi}{6},\ \pm\frac{5\pi}{6}$	(A1)		
	Writes coordinates of points.	$\left(\frac{a}{\sqrt{2}},\pm\frac{\pi}{6}\right),\ \left(\frac{a}{\sqrt{2}},\pm\frac{5\pi}{6}\right)$ (Award final A1 if *r* found but result not written out.)	(A1)	(7)	**[14]**

Qu No	Commentary	Solution	Marks	M	Total
11	**OR**				
	Differentiates once.	$\frac{\mathrm{d}y}{\mathrm{d}x} = z + x\frac{\mathrm{d}z}{\mathrm{d}x}$	B1		
	Differentiates again.	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\frac{\mathrm{d}z}{\mathrm{d}x} + x\frac{\mathrm{d}^2z}{\mathrm{d}x^2}$	B1		
	Substitutes.	$\frac{\mathrm{d}^2z}{\mathrm{d}x^2} + \left(\frac{2}{x} + 6 - \frac{2}{x}\right)\frac{\mathrm{d}z}{\mathrm{d}x} + \left(\frac{6z}{x} - \frac{2z}{x^2} + 9z - \frac{6z}{x} + \frac{2z}{x^2}\right)$ $= 169\sin 2x$	M1		
	Obtains result	$\Rightarrow \frac{\mathrm{d}2z}{\mathrm{d}x2} + 6\frac{\mathrm{d}z}{\mathrm{d}x} + 9z = 169\sin 2x$ (AG)	A1	4	
		(Mark similarly if substitution is rearranged to $z = \frac{y}{x}$.)			
	Finds CF.	$m^2 + 6m + 9 = (m+3)^2 = 0 \Rightarrow m = -3$ CF: $A\mathrm{e}^{-3x} + Bx\mathrm{e}^{-3x}$	M1 A1		
	Finds PI.	PI: $y = p\sin 2x + q\cos 2x$ $y' = 2p\cos 2x - 2q\sin 2x$ $y'' = -4p\sin 2x - 4q\cos 2x$	M1		
		$5p - 12q = 169$ $12p + 5q = 0$ $\Rightarrow p = 5$ and $q = -12$	M1 A1		
	Finds GS.	GS: $z = A\mathrm{e}^{-3x} + Bx\mathrm{e}^{-3x} + 5\sin 2x - 12\cos 2x$	A1		
	Evaluates coefficients from initial conditions.	$-10 = A - 12 \Rightarrow A = 2$ $z' = -6\mathrm{e}^{-3x} + B\mathrm{e}^{-3x} - 3Bx\mathrm{e}^{-3x} + 10\cos 2x + 24\sin 2x$ $5 = -6 + B + 10 \Rightarrow B = 1$	B1 M1 A1		
	Finds particular solution.	$z = 2\mathrm{e}^{-3x} + x\mathrm{e}^{-3x} + 5\sin 2x - 12\cos 2x$ $\therefore y = 2x\mathrm{e}^{-3x} + x^2\mathrm{e}^{-3x} + 5x\sin 2x - 12x\cos 2x$	A1	10	[14]